

Putnam Problems — Prof. Madras

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Here are some problems that one can solve by “chipping away” with a (possibly lengthy) sequence of small steps. These could be good questions for getting part marks!

2017-A1. Let S be the smallest set such that

- (a) 2 is in S ,
- (b) n is in S whenever n^2 is in S , and
- (c) $(n + 5)^2$ is in S whenever n is in S .

Which positive integers are not in S ?

2020-A1. How many positive integers N satisfy all of the following three conditions?

- (a) N is divisible by 2020.
- (b) N has at most 2020 digits.
- (c) The decimal digits of N are a string of consecutive ones followed by a string of consecutive zeros.

2012-B1. Let S be the class of functions from $[0, \infty)$ to $[0, \infty)$ that satisfy:

- (i) The functions $f_1(x) = e^x - 1$ and $f_2(x) = \ln(x + 1)$ are in S .
- (ii) If $f(x)$ and $g(x)$ are in S , then the functions $f(x) + g(x)$ and $f(g(x))$ are in S .
- (iii) If $f(x)$ and $g(x)$ are in S and $f(x) \geq g(x)$ for all $x \geq 0$, then the function $f(x) - g(x)$ is in S .

Prove that if $f(x)$ and $g(x)$ are in S , then the function $f(x)g(x)$ is also in S .