Putnam Problems — Prof. Madras

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2005-B1. Find a nonzero polynomial $P(x, y)$ such that $P(\lfloor a \rfloor, \lfloor 2a \rfloor) = 0$ for all real numbers a. (Note: $\lfloor \nu \rfloor$ is the greatest integer less than or equal to ν.)

2023-A1. For a positive integer n, let

$$f_n(x) = \cos(x)\cos(2x)\cos(3x)\cdots\cos(nx).$$

Find the smallest n such that $|f_n''(0)| > 2023$.

2016-B1. Let $x_0, x_1, x_2, \ldots$ be the sequence such that $x_0 = 1$ and for $n \geq 0$

$$x_{n+1} = \ln(e^{x_n} - x_n)$$

(as usual, the function ln is the natural logarithm). Show that the infinite series

$$x_0 + x_1 + x_2 + \cdots$$

converges and find its sum.