Putnam Notes — Prof. Madras

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On September 23, we mentioned the *Pigeonhole Principle:*

> If n items are put into m boxes, and if $n > m$, then at least one box must contain more than one item.

Here are some problems that can be solved using the Pigeonhole Principle.

PP1. (a) Show that in any set of three integers $\{x_1, x_2, x_3\}$, there must be two of them whose average is an integer.
(b) Show that in any set of five ordered pairs of integers $\{(x_i, y_i), i = 1, 2, 3, 4, 5\}$. there must be two of them whose average is another ordered pair of integers.

PP2. You have a 3×7 grid of squares. Each square is coloured red or blue. Show that there is a sub-rectangle of the grid whose four corner squares all have the same colour.
(*Suggestion:* Think about the number of possible coloured columns. Why would this question be easier if we changed 3×7 to 3×9?)

PP3. Prove that every set of ten two-digit natural numbers has two disjoint subsets with the same sum of elements.