Putnam Problems — Prof. Madras

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We shall work on the following from the "Easy Putnam Problems" file. One of them is indeed very easy in hindsight, but it is not immediately obvious! The others are all pretty accessible.

2011–A1. Given a positive integer n, what is the largest k such that the numbers $1, 2, \ldots n$ can be put into k boxes so that the sum of the numbers in each box is the same? [When $n = 8$, the example $\{1, 2, 3, 6\}$, $\{4, 8\}$, $\{5, 7\}$ shows that the largest k is *at least* 3.]

2004–A1. Basketball star Shanille O'Keal's team statistician keeps track of the number, $S(N)$, of successful free throws she has made in her first N attempts of the season. Early in the season, $S(N)$ was less that 80% of N, but by the end of the season, $S(N)$ was more than 80% of N. Was there necessarily a moment in between when $S(N)$ was exactly 80% of N?

1988–B1. A *composite* (positive integer) is a product ab with a and b not necessarily distinct integers in $\{2, 3, 4, \ldots\}$. Show that every composite is expressible as $xy + xz + yz + 1$, with x, y, and z positive integers.

2009–A1. Let f be a real-valued function on the plane such that for every square $ABCD$ in the plane, $f(A) + f(B) + f(C) + f(D) = 0$. Does it follow that $f(P) = 0$ for all points P in the plane?